

Eavesdropping of quantum communication from a non-inertial frame

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We introduce a relativistic version of quantum encryption protocol by considering two inertial observers who wish to securely transmit quantum information encoded in a free scalar quantum field state forming Minkowski particles. In a non-relativistic setting a certain amount of shared classical resources is necessary to perfectly encrypt the state. We show that in the case of a uniformly accelerated eavesdropper the communicating parties need to share (asymptotically in the limit of infinite acceleration) just half of the classical resources.

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Only relatively a small fraction of works on quantum information theory studies its various aspects in a relativistic setting [1]. The violation of Bell's inequality for a quantum entangled state shared by two observers who are not at rest with respect to each other in special [2] or general relativity [3] attracted a bit more attention. Also, it was observed that the entanglement is a quantity depending on a relative acceleration of one of the observers who, before being accelerated, shared a maximally entangled bosonic [4] or fermionic pair [5]. In the latter it was found that the entanglement does not vanish even in the limit of infinite acceleration. The physical effect behind the scene responsible for this behavior is the Unruh effect [6] which can be generalized into an arbitrary spacetime dimension by the thermalization theorem [13]. As a seminal example of its consequences on the effectivity of quantum information protocols in a relativistic setting let us recall a relativistic version of the quantum teleportation protocol [7]. In this case Alice is a stationary observer sharing a maximally entangled state with Bob in the same frame. Later, Bob uniformly accelerates with respect to Alice and it is found that the fidelity of an input and output state is a decreasing function of Bob's acceleration. Due to the Unruh effect, Bob has no longer access to the maximally entangled state and the higher acceleration he perceives the less entangled state is effectively shared by both participants. Consequently, the reliability of the teleportation is reduced. Therefore, this quite different behavior from the 'standard case' leads us to the revision of the other quantum communication protocols.

In this paper we consider relativistic effects in a cryptographic protocol known as Private Quantum Channel (PQC). We are interested in learning how can relativistic effects be used to relax the assumptions on classical resources needed for the perfect encryption of a quantum state. It is known that for an eavesdropping in the same frame where the communication takes place two bits of shared classical information are necessary for the secure transmission of the state. We have shown that the amount of the shared classical resources is reduced if the eavesdropper (Eve) is in a relative uniform acceleration with respect to inertial Alice and Bob. In the limit of infinite acceleration, the classical cost of the secure encryption is reduced to one half. The infinite acceleration limit corresponds to a situation in which freely falling Alice and Bob (into a black hole) transmit information while Eve escaping the fall tries to tap the communication. Note that in the following analysis we will encrypt qubits but our results can be directly generalized to the case of d -level states (qudits). On the other hand, the encryption of states with infinitely many degrees of freedom (so called continuous-variable quantum states) requires much more caution even in the non-relativistic case [9].

The structure of the paper is the following. In Sec. I we remind the concept of Minkowski (Rindler) particles as a free massless scalar field given by the quantization of a superposition of plane waves forming wave packets in Minkowski (Rindler) spacetime. We will employ an effect similar to the Unruh effect (derived for the single photon field excitations), that is, a decomposition of the Minkowski one-particle state into the many-particle Rindler state occupying two causally disconnected regions. In Sec. II we recall some of the basic properties of PQCs together with the justification of their use. The main part comes in Sec. III where we derive an upper bound on information which a non-inertial eavesdropper, in the limit of infinite acceleration, is able to extract from partially encrypted communication being in progress between two legal inertial observers.

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I. TRANSFORMATION OF MINKOWSKI PARTICLES INTO A NON-INERTIAL FRAME

The wave packets, whose properties will be employed, were studied in detail in [12] (and originally come from [14]) but for more general approach to the particle/wave packet concept in QFT we refer to Takagi's noticeable earlier paper [13]. The used wave packets have the usual form of a superposition of plane waves in both spacetimes modulated by Fourier coefficients with a simple structure. The quantization was performed for a free massless scalar bosonic field and the resulting objects in Minkowski spacetime are known as Minkowski particles (and similarly for Rindler spacetime). Such an object is spatially and temporarily localized and thus allows us to talk about preparation, manipulation and measurement as known from non-relativistic quantum mechanics. This is not possible with a single field excitation (Fock state) as understood by QFT (similar approach with a different kind of quantized wave packets was quite recently used for predicting of a decoherence effect of an entangled pair with one of the qubits perceiving changes in the gravitational field [15]). How could a communication with Minkowski particle look like? This is in fact the question of the information encoding, i.e. defining a logical qubit communication basis. As we will see, the choice of the encoding has an impact on the calculation using the field transformation from Minkowski to Rindler spacetime.

Let the Minkowski observers' basis be composed of two one-Minkowski particles both in two orthogonal spatial modes (here indexed as 1,2). This is known as the dual rail encoding used, for example, in linear-optical quantum computing schemes [16] or an experimental realization of quantum teleportation protocol [17]. Then, we can write our logical qubits as

$$\{|\mathbb{O}\rangle \equiv |0\rangle_1^{\mathcal{M}} |1\rangle_2^{\mathcal{M}}, |\mathbb{I}\rangle \equiv |1\rangle_1^{\mathcal{M}} |0\rangle_2^{\mathcal{M}}\}, \quad (1)$$

where $|0\rangle, |1\rangle$ is the Minkowski vacuum and the Minkowski one-particle, respectively. Similarly as for the quantization of Fock states the field is quantized in both Minkowski and Rindler spacetime and the corresponding creation and annihilation operators (before the quantization being the mode expansion coefficients) are defined. Using the Klein-Gordon inner product these two sets of operators are related and the creation and annihilation operators from one space are expressed as linear combinations of those from the other space. The coefficients in these combinations are known as the Bogoliubov particle coefficients yielding [12] the transformed Minkowski vacuum (the Rindler vacuum)

$$|0\rangle_1^{\mathcal{M}} = \prod_{k,l} \frac{1}{\cosh r_k} \sum_{g=0}^{\infty} \tanh^g r_k |g-k, l\rangle_{1L} |g, l\rangle_{1R}, \quad (2)$$

where $\cosh r_k = (1 - e^{-2\pi\omega_k c/a})^{-1/2}$, $\tanh r_k = e^{-\pi\omega_k c/a}$ with a the Rindler observer's proper acceleration and e.g. $|g, l\rangle_{1R}$ is the Rindler g -particle state with integer mode numbers k, l characterizing the particle in Rindler spacetime in spatial mode 1 and the right wedge. The Minkowski one-particle state as seen by the Rindler observer has the form [12]

$$|1_{\tilde{k}\tilde{l}}\rangle_1^{\mathcal{M}} = \left[\prod_{\substack{k \neq k_t \\ l \neq l_t}} \frac{1}{\cosh r_k} \sum_{p=0}^{\infty} \tanh^p r_k |p-k, l\rangle_{1L} |p, l\rangle_{1R} \right] \times \frac{1}{\cosh^2 r_{k_t}} \sum_{q=0}^{\infty} \sqrt{1+q} \tanh^q r_{k_t} |(1+q)-k_t, l_t\rangle_{1L} |q, k_t, l_t\rangle_{1R}, \quad (3)$$

where the indexes with tildes indicate the particles in Minkowski space and k_t, l_t are so called equivalent modes in Rindler spacetime [12] corresponding to the indexes $\tilde{k}\tilde{l}$. As usual, the Rindler observer has access just to one of the wedges (in our case to the right one). Generally, we can see the glimpse of similarity to the transformation of ordinary Fock states.

Let us see how a density matrix in the dual-rail representation transforms as a whole. Taking the corresponding products of Eqs. (3) and (2) and tracing over the left wedge the matrix elements acquire the form of infinite-dimensional matrices. First, for the diagonal elements we have

$$\begin{aligned} \text{Tr}_L [|\mathbb{O}\rangle\langle\mathbb{O}|] &= \prod_{k,l} \frac{1}{\cosh^2 r_k} \sum_{g=0}^{\infty} \tanh^{2g} r_k |g, l\rangle\langle g, l|_1 \\ &\times \prod_{\substack{k \neq k_t \\ l \neq l_t}} \frac{1}{\cosh^2 r_k} \sum_{p=0}^{\infty} \tanh^{2p} r_k |p, l\rangle\langle p, l|_2 \times \frac{1}{\cosh^4 r_{k_t}} \sum_{q=0}^{\infty} (1+q) \tanh^{2q} r_{k_t} |q, k_t, l_t\rangle\langle q, k_t, l_t|_2, \end{aligned} \quad (4)$$

and similarly for $\text{Tr}_L [|\mathbb{I}\rangle\langle\mathbb{I}|]$. We see that both matrices are diagonal due to the orthogonality relations valid for k, l

and the spatial modes. After tracing over the left wedge of the transformed off-diagonal matrix elements we get

$$\begin{aligned}
\text{Tr}_L [|\mathbb{O}\rangle\langle\mathbb{I}|] &= \prod_{\substack{k \neq k_t \\ l \neq l_t}} \frac{1}{\cosh^2 r_k} \sum_{p=0}^{\infty} \tanh^{2p} r_k |p_{k,l}\rangle\langle p_{k,l}|_1 \times \frac{1}{\cosh^3 r_{k_t}} \sum_{q=0}^{\infty} \sqrt{1+q} \tanh^{2q} r_{k_t} |(1+q)_{k_t,l_t}\rangle\langle q_{k_t,l_t}|_1 \\
&\times \prod_{\substack{k \neq k_t \\ l \neq l_t}} \frac{1}{\cosh^2 r_k} \sum_{f=0}^{\infty} \tanh^{2f} r_k |f_{k,l}\rangle\langle f_{k,l}|_2 \times \frac{1}{\cosh^3 r_{k_t}} \sum_{g=0}^{\infty} \sqrt{1+g} \tanh^{2g} r_{k_t} |g_{k_t,l_t}\rangle\langle (1+g)_{k_t,l_t}|_2 \\
&= (\text{Tr}_L [|\mathbb{I}\rangle\langle\mathbb{O}|])^T.
\end{aligned} \tag{5}$$

Thus, both off-diagonal elements from Minkowski spacetime become off-diagonal matrices in Rindler spacetime and a general Minkowski density matrix (in the dual-rail representation) is symmetrical and three-diagonal after the transformation.

II. PERFECT AND PARTIAL QUANTUM STATE ENCRYPTION

Having introduced the concept of localized objects both in Minkowski and Rindler spacetime we can start generalizing PQC into the relativistic setting. Let Alice and Bob be two inertial observers in Minkowski spacetime who share a random and private string of bits – key and are connected through a quantum channel. The bits of the key index unitary operations (to be specified) are defined in a space spanned by the dual rail basis vectors (1). Hence, Alice receives/prepares an arbitrary and unknown quantum state (generally a mixed state written in the dual rail basis (1)) and applies one of the unitaries dictated by the key. She sends the modified state down the channel to Bob who makes the inverse unitary operation. The task for Alice and Bob is to securely transmit the input (unencrypted) state and to prevent from a potential eavesdropping by Eve. She has full access to the quantum channel and is able to construct any generalized measurement (POVM) that could give her some information about the state. In our case, we suppose that Eve is a non-inertial observer so let us investigate the consequences for the protocol security.

The theory of PQCs learns [8] that for any qubit Ξ holds

$$\Xi \mapsto \frac{1}{4} \sum_{i=1}^4 \sigma_i \Xi \sigma_i^\dagger = \frac{\mathbb{1}}{2} = \varrho, \tag{6}$$

where the encryption set composed of the Pauli matrices $\{\sigma_i\} \Rightarrow \{\mathbb{1}, \sigma_X, \sigma_Y, \sigma_Z\}$ is the usual (but non-unique) choice for the encryption procedure. No matter the form of Ξ someone without the knowledge which operation for the encryption σ_i was used has no chance to get any information about the state since in the identity density matrix there is no information on the input state. This is called a perfect encryption. We also see that necessary (and also sufficient [8]) length of the shared key for the encryption of one qubit are two bits. On the other hand, if we don't adhere to the rule of not using less than a two-bit key and use, for instance σ_1, σ_2 , i.e. just a one-bit key, we get a partially encrypted qubit

$$\Xi \mapsto \frac{1}{2} \sum_{j=1}^2 \sigma_j \Xi \sigma_j^\dagger = \frac{1}{2} \begin{pmatrix} 1 & f(\Xi_{12}, \Xi_{21}) \\ f(\Xi_{12}, \Xi_{21}) & 1 \end{pmatrix} = \tilde{\varrho}. \tag{7}$$

Hence, in the off-diagonal terms of $\tilde{\varrho}$ there is some hidden information about Ξ and the state is not secured. So, assuming the partial encryption a possible inertial eavesdropper would apparently have a chance to get some information on the input state. But how is it with Eve as the non-inertial eavesdropper? Before answering this question let us make a small detour and say something general about the PQC applicability.

Obviously, PQC is meaningful whenever an eavesdropper can get some information about density matrix elements by any kind of generalized measurement. Into this situation falls (a) the case when we have several copies of an unknown state (or generally classically correlated input and thus we can make a quantum state estimation) or (b) the case when we have one copy of an arbitrary mixed state $\Xi = \sum p_i \Xi_i$ with given a priori probabilities p_i and we implement one of possible discrimination techniques such as unambiguous state discrimination (if applicable), minimum error discrimination or their advanced combinations [18]. On the contrary, if one has one copy of an unknown or even known state then the encryption is unnecessary. In the first case the state is already encrypted (an unknown qubit is seen as a mixture of all states in the Bloch Sphere that is proportional to a maximally mixed state) and in the latter case one just needs to send classical information about the state preparation and a high-priced quantum channel is not

required for the state transmission (the question of information load tradeoff between classical and quantum channel is not interesting for us now).

For later purposes, let us shortly discuss how the leakage of information of an encrypted state can be measured. The extractable information is quantified by the accessible information which is a maximization over all possible POVMs. However, finding the optimal POVM is a rather difficult task and analytical solutions are known only for few cases, mostly under some kind of symmetry [10]. In this light, the quantity we will use is the Holevo information [11]

$$\chi(\{p_i, \varrho_i\}) = S(\varrho) - \sum_{i=1}^n p_i S(\varrho_i) \quad (8)$$

bounding the accessible information from above, where $\varrho = \sum p_i \varrho_i$ and $S(\varrho) = -\text{Tr} [\varrho \log_2 \varrho]$ is the von Neumann entropy. The Holevo bound is more feasible to calculate than the accessible information but it is not generally a tight bound. Fortunately, in the case we are investigating it is so. To see it, let us assume that an input qubit is fully encrypted

$$\Xi = \sum_{i=1}^n p_i \Xi_i \xrightarrow{PQC} \sum_{i=1}^n p_i \underbrace{\frac{1}{4} \sum_j^4 \sigma_j \Xi_i \sigma_j^\dagger}_{\varrho_i \propto \mathbb{1}} = \varrho (\propto \mathbb{1}). \quad (9)$$

Then, for the best possible discrimination of the states before encryption (Ξ_i) Eve has to distinguish among ϱ_i because the non-encrypted states were transformed onto this 'encoding'. Inserting the encoding into (8) we get $\chi(\{p_i, \varrho_i\}) = 0$. Of course, after the encryption Eve may attempt to distinguish any encoding at her will (and the Holevo bound does not need to be zero) but whatever discrimination strategy she chooses at the end of the day she always gets zero information about the occurrence of the states Ξ_i before the encryption [8].

III. INFORMATION GAIN OF AN ACCELERATED EAVESDROPPER

The purpose of this main part is to show that in the limit of infinite acceleration an eavesdropper (Eve as the Rindler observer) cannot get any information from just a partially encrypted qubit. First, we show that in the infinite limit the Holevo bound for a perfectly encrypted qubit is the same as a partially encrypted one. Second, despite the Holevo bound not being a tight upper bound for the accessible information we show that the accessible information available to the accelerated eavesdropper tapping a partially encrypted qubit is zero as well. We may write

$$\begin{aligned} |\chi(\{p_i, \varrho_i\}) - \chi(\{\tilde{p}_i, \tilde{\varrho}_i\})| &= \left| S(\varrho) - S(\tilde{\varrho}) + \sum_{i=1}^n \tilde{p}_i S(\tilde{\varrho}_i) - \sum_{i=1}^n p_i S(\varrho_i) \right| \\ &\leq |S(\tilde{\varrho}) - S(\varrho)| + \left| \sum_{i=1}^n \tilde{p}_i S(\tilde{\varrho}_i) - \sum_{i=1}^n p_i S(\varrho_i) \right| \quad \text{by the triangle inequality.} \end{aligned} \quad (10)$$

Next, in the spirit of Eq. (9) we use $S(\varrho_i) = S(\varrho)$ and prove the convergence of the second summand in the second row of Eq. (10). Later, using the same equality, we show that the convergence of the first summand follows from a simple modification of the proof of the convergence of the second summand. It is a special case simply by putting $n = 1$ ($\tilde{p}_i = 1$), i.e. omitting the sum over n in all the derivations which follow.

Hence, rewriting the second summand in Eq. (10) and generating its Taylor series about any (identical) nonzero point we get after the rearranging [20]

$$\sum_{j=1}^{\infty} \left(\sum_{i=1}^n \tilde{p}_i \tilde{\lambda}_{ij} \log_2 \tilde{\lambda}_{ij} - \lambda_j \log_2 \lambda_j \right) = \sum_{j=1}^{\infty} \left(\sum_{k=0}^{\infty} \left(b_k \sum_{i=1}^n \tilde{p}_i \tilde{\lambda}_{ij}^k - \lambda_j^k \right) \right) = \sum_{k=0}^{\infty} \left(b_k \sum_{i=1}^n \tilde{p}_i \text{Tr} [\tilde{\varrho}_i^k] - \text{Tr} [\varrho^k] \right), \quad (11)$$

where $\tilde{\lambda}_{ij}, \lambda_j$ is the j -th eigenvalue of $\tilde{\varrho}_i$ and ϱ , respectively, and $\sum_{j=1}^{\infty} \tilde{\lambda}_{ij}^k = \text{Tr} [\tilde{\varrho}_i^k]$, $\sum_{j=1}^{\infty} \lambda_j^k = \text{Tr} [\varrho^k]$ was used. b_k are the Taylor expansion coefficients and are set up to be identical for all expanded functions in our case [20]. Eq. (11) is trivially equal to zero if $k = 0$, the same holds for $k = 1$ ($\tilde{\varrho}_i, \varrho$ are density matrices and $\sum \tilde{p}_i = 1$). For

$k = 2$ we will show its asymptotic convergence to zero in Rindler space by a direct calculation. This will be a starting point for proving the convergence for $k > 2$. First, we set up the following notation

$$a_q = \frac{\tanh^{2q} r}{\cosh^4 r} (1 + q) \quad A_q = \frac{\tanh^{2q} r}{\cosh^3 r} \sqrt{1 + q} \quad (12)$$

($r \in \mathbb{R}^+$) where a_q are the elements of Eq. (4) forming the diagonal of Eqs. (6) and (7) in Rindler spacetime. Similarly, the coefficients A_q coming from Eq. (5) are the off-diagonal elements of Eq. (7) in Rindler spacetime. The off-diagonal coefficients of the matrices $\tilde{\varrho}_i$ (coming from Eq. (7) for $\Xi = \sum p_i \Xi_i$) are the scalar functions $f_i(\Xi_{12}, \Xi_{21})$ independent of the acceleration and do not transform. Also, note that we work just with one mode k, l [21]. Next, for the upcoming considerations we need to know that $a_q > a_{q+1}$, $\lim_{r \rightarrow \infty} \sum_{q=0}^{\infty} a_q^2 = 0$, $0 \leq a_q \leq 1$ and the same holds for A_q . Writing the square of $\tilde{\varrho}_i$ we get a five-diagonal infinite-dimensional matrix where on the diagonals we generally find sequences of the type $\sum a_q a_p$, $\sum A_q A_p$ and $\sum a_q A_p$ [22]. Due to the properties of a_q, A_q all of them can be bounded from above by putting $p = q$ thus the sequences (and of course their elements) converge to zero for $r \rightarrow \infty$.

To continue let us consider the following inequalities

$$\sum_{i=1}^n \tilde{p}_i \text{Tr} [\tilde{\varrho}_i^k] - \text{Tr} [\varrho^k] \leq \sum_{i=1}^n \text{Tr} [\tilde{\varrho}_i^k] - \text{Tr} [\varrho^k] \leq \text{Tr} \left[\left(\sum_{i=1}^n \tilde{\varrho}_i + \varrho \right)^k \right] \leq \text{Tr} [(n+1)^k \tilde{\varrho}_{\max}^k], \quad (13)$$

where, as stated, all $\tilde{\varrho}_i$ have the same form as Eq. (7) but with different scalar functions $f_i(\Xi_{12}, \Xi_{21})$. Thus, the maximum is taken over n scalar functions. The last inequality in Eq. (13) does not hold in general but the structure of $\tilde{\varrho}_i, \varrho$ allows us to do it (recall that we are in Rindler spacetime where ϱ is a diagonal density matrix and $\tilde{\varrho}_i$ are three-diagonal density matrices with the main diagonal equal to the diagonal of ϱ). Then

$$\lim_{r \rightarrow \infty} \left(\sum_{i=1}^n \text{Tr} [\tilde{\varrho}_i^k] - \text{Tr} [\varrho^k] \right) \leq \lim_{r \rightarrow \infty} \text{Tr} \left[(m \tilde{\varrho}_{\max}^2)^{k/2} \right] = m \text{Tr} \left[\left(\lim_{r \rightarrow \infty} \tilde{\varrho}_{\max}^2 \right)^{k'} \right], \quad (14)$$

where $m = (n+1)^2$, $k' = k/2$ (note that for the convergence of the first summand in Eq. (10) we have $m = 1$ and thus there is no need for the maximization over n). We use some basic limit properties in the last equality with $\lim_{r \rightarrow \infty} \tilde{\varrho}_{\max}^2$ meaning the limit of *every* element of the matrix $\tilde{\varrho}_{\max}^2$. Since for $r \rightarrow \infty$ the limit converges to zero its k' -th matrix power converge as well thus proving the convergence of Eq. (11) to zero. Consequently, we bounded the Holevo information difference (10) from above.

Since the Holevo bound is not a tight bound on the accessible information it remains one step more and we follow up with the discussion from the end of Sec. II. From Eq. (10) we know that the difference goes asymptotically to zero. Now we choose the encoding $\{p_i, \varrho_i\}$ of the perfect PQC such that $\chi(\{p_i, \varrho_i\}) = 0$ both in Minkowski and Rindler spacetime. In fact it corresponds to Eve's choice of the discrimination strategy in Rindler spacetime. Although we cannot dictate Eve what to do, in this case, due to the properties of perfect PQC, any tapping strategy she is able to do gives her no information. Then, from the convergence above together with Eq. (10) it follows that the Rindler observer (Eve) cannot get any information even from the partially encrypted quantum state.

IV. CONCLUSIONS

We addressed the question of the role of the Unruh effect on security of quantum communication. We considered the setting where two honest parties are at rest (Minkowski observers) and an eavesdropper is uniformly accelerated with respect to them (Rindler observer). The Minkowski observers established PQC where logical qubits are represented by a free scalar field in the form of so-called Minkowski particles considering the dual-rail encoding. The legal parties did not satisfy the security requirements and they encrypted a qubit with just one bit of a classical key instead of two bits. Since this is a necessary and sufficient condition for the perfect encryption, it would generally lead to a leakage of some information about the qubit. However, we have shown that in the limit of infinite acceleration this partial encryption is sufficient to secure the qubit and, thus, it is prevented from getting any information into the hands of an eavesdropper.

In other words, we have shown that the Unruh effect makes the communication noisy in the direction where the eavesdropper in a non-inertial frame cannot distinguish between a perfectly and partially encrypted qubit. The important aspect of the above conclusion is that we did not consider problematic single photon excitations (Fock states) but temporarily and spatially localized objects both in Minkowski and Rindler spacetime (Minkowski and Rindler particles, respectively). The problem of Fock states lies in their delocalization making difficult to talk about

the preparation or manipulation (considering the production of Fock states in a realistic cavity which is subsequently accelerated brings even more problems). This is the first step toward the description of a more natural and general setup where both the legal participants and Eve are differently accelerated and the objective would be to enumerate the leaked information not only in the asymptotic case.

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 - [20] We are basically looking for the Taylor series of a multivariate function which is in the form $g(x_1, \dots, x_{n+1}) = \sum_{i=1}^{n+1} g(x_i)$. Inserting this function into the prescription for the Taylor series [19] we find that it corresponds to the summation of the Taylor decompositions of particular $g(x_i)$ (generally about different points). In our case we put everywhere the same point.
 - [21] It is sufficient to work with just one mode because the matrix elements in Rindler space are mode products of sums of particular modes. Proving the convergence of the sum just for one particular mode and using $0 \leq a_q, A_q \leq 1$ we show that the whole product converges.
 - [22] Except a_q, A_q we can also find there $\sum_q \tanh^{2q} r / \cosh^2 r$ for which convergence is easy to see.